RESEARCH ON DETERMINING THE EXPRESSION OF HARDNESS VARIATION OF MATERIALS USED IN SHOE HEEL MANUFACTURING DEPENDING ON THE MEDIO-LATERAL COMPONENT, Fy, OF THE GROUND REACTION FORCE

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This study was aimed at obtaining an empirical relationship in order to use hardness of materials in constructing a custom heel depending on the medio-lateral component, Fy, of the ground reaction force. Custom heeled orthopedic footwear is designed for people with no structural or functional abnormalities in the lower limb, but who value comfort in wearing shoes to the detriment of other aspects related to heel height or shape. The three components of the ground reaction force, namely: the vertical component, Fz, the antero-posterior component, Fx, and the medio-lateral component, Fy, upon ground contact, were measured with AMTI's AccuGait force plate, using the NetForce component, and they were analyzed using the BioAnalysis module. Tests were conducted to analyze materials with various hardness values used in the construction of a custom heel. Thus, after determining the relation, depending on the desired value of the medio-lateral component, Fy, of the ground reaction force, the hardness required for the construction of the custom heel is obtained.

Keywords: hardness, custom heel, ground reaction

INTRODUCTION

Manufacturing custom heeled shoes in order to reduce the medio-lateral component of the ground reaction force, involves the use of materials with different hardness, depending on gait style, walking surfaces and type of shoe, so as to provide the required comfort during walking. Viscoelastic materials for the heel and sole have been widely used since the 1980s, in order to ameliorate or prevent certain diseases of the foot joints, plantar fasciitis, sprains, stress fractures, etc., due to the property of reducing the size of the impact forces upon heel contact with the ground (Whittle, 1996).

Previous studies have shown that attenuation of ground reaction force is a constant concern in footwear manufacturing (Clarke *et al.*, 1983; Logan *et al.*, 2010; Cavanagh *et al.*, 1979).

Experiments performed yielded a database that was used in manufacturing custom heel shoes with the aim to reduce the medio-lateral component of the ground reaction force. The proposed solutions have been based on the development of custom heels of different materials with different hardness for different types of shoes. The three components of the ground reaction force, namely: the vertical component, Fz, the antero-posterior component, Fx, and the medio-lateral component, Fy, upon ground contact, were measured with AMTI's AccuGait force plate, using the NetForce component, and they were analyzed using the BioAnalysis module (Vasilescu *et al.*, 2011; 2012; 2013).

For the design of custom heeled orthopedic footwear in order to reduce the medio-lateral component of the ground reaction force, the possibility of obtaining an empirical relationship that allows the determination of hardness of materials used in heel construction depending on the medio-lateral force, Fy, was analyzed.

METHOD

In order to obtain an unknown function that describes a data set exhibiting a curvilinear behavior, the most common method is non-linear regression. This method involves the generation of new variables in that data set. Thus, once these variables are set, the unknown function (with curvilinear appearance) in the initial state can be expressed as a linear function by the new variables (Baker, 2008; Geaghan, 2012; Seber and Wild, 1989; Meade and Islam, 1995; Schittkowski, 2002; Bethea *et al.*, 1985; Oosterbaan *et al.*, 1990).

To illustrate the above, we may use two non-linear equations encountered in technique (Baker, 2008):

Equation	Interpretation	Linearized form
$Y = Ae^{bX}u$	Y function has a variation (increase/decrease) with a rate dependent on b	$ln(Y) = ln(A) + bX + ln(u)$
$Y = AX^{b}u$	Elasticity of the Y function in relation to X is a b constant	$ln(Y) = ln(A) + b \cdot ln(X) + ln(u)$

In the first example, the exponential growth depends on the b term while the u term models the error (deviation).

Using logarithms on both sides of the equation yields:

$$ln(Y) = ln(A) + bX + ln(u) \quad (1)$$

It is noticed that, although the equation contains logarithms, the terms are linearly dependent on each other. In this form, we may write – by changing the variable:

$$y = a + bX + eroare \quad (2)$$

Thus, by creating a new variable, the natural logarithm of Y, we can use the method of least squares to obtain a regression relationship.

We can proceed similarly with the second equation:

$$Y = AX^{b}u \quad (3)$$

It is specific to processes with constant elasticity (the term elasticity is used broadly, without necessarily referring to mechanical elasticity). Proceeding to apply logarithms to the two sides of the equation, we get a linearized form:

$$ln(Y) = ln(A) + b \cdot ln(X) + ln(u) \quad (4)$$

The above examples are a way of approaching this problem: by empirically estimating the shape (appearance) of the desired equation we can then linearize the said equation in order to apply the method of least squares. There is a whole series of types of equations with various shapes which may be used as initial estimations. The initial equation is usually chosen by evaluating the generic shape described by the set of experimental data; this practice is, however, a simplistic one and does not reflect the complexity of this mathematical instrument.

Thus, by understanding the nature of the physical phenomenon, a shape of the unknown equation can be drawn. Then, by iterative variation of its various constants and parameters, we can obtain a physical-mathematical model that fits experimental data. It is noteworthy that the importance of choosing the shape of the empirical equation may be higher than the otherwise intuitive principle of choosing an equation that leads to a minimum deviation.

RESULTS AND DISCUSSIONS

The utility of a well constructed equation is that, by adding new experimental data, eventually, a theoretical depth of the phenomenon in question can be reached, while by choosing a polynomial equation of degree n, this is more difficult. Also, by using a carefully constructed equation, the limits or restrictions useful in defining the studied phenomenon can be drawn.

It is noted that there is compatibility between the analytical function shaped like a third-order polynomial equation and the graph based on the experimental data set. The polynomial function may have a higher degree, however, it introduces an increasingly higher number of constants that are only desirable if they lead to more accurate approximations.

The dependence of medio-lateral component, Fy, on hardness can be expressed by the following equation (5):

$$y = 0.0005x^3 - 0.1018x^2 + 6.6289x - 1\ 3 \qquad (5)$$

Figure 1 presents the evolution of the medio-lateral component, Fy, depending on hardness.

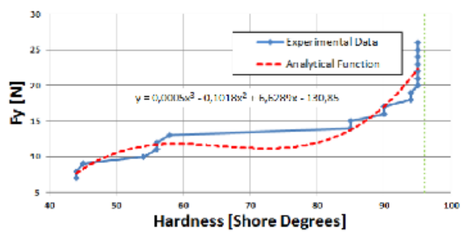

Figure 1. Evolution of the medio-lateral component depending on hardness

The dependence of hardness on the medio-lateral component, Fy, can be expressed by the following equation:

$$Hardness = -4 \cdot 10^{-5} x^5 + 0.0073x^4 - 0.3949x^3 + 8.6715x^2 - 77.032x + 278.1 \qquad (6)$$

It is noted that the dependence is of the 5th order, with 6 empirical terms - virtually representing, in turn, functions related to the physical process. Although this relationship has a less intuitive physical interpretation, it fulfills a very precise purpose because it offers the possibility of linking the two physical parameters.

Thus, by entering the desired value of the medio-lateral component, Fy, of the ground reaction force, the hardness required for manufacturing a custom heel is obtained.

The evolution of hardness depending on the medio-lateral component is shown in Figure 2.

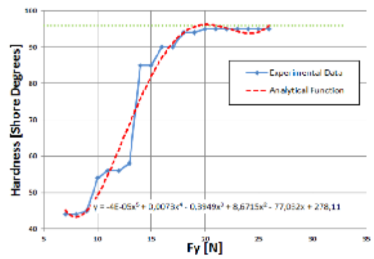

Figure 2. Evolution of hardness depending on the medio-lateral component, Fy

Another approach is that of using an equation similar to the Boltzmann sigmoidal equation:

$$Y = LowerLimit + (UpperLimit - LowerLimit) / \{1 + \exp[(a - X) / IntermediarySlope]\} \quad (7)$$

In this case we can express the physical process by introducing values obtained for the four variables of the function, using the following equation (hardness depending on Fy) (8):

$$Hardness = 45.9 + \frac{48.87}{1 + e^{(13.36 - x)/1.138}} \quad (8)$$

Figure 3 shows correlation between the empiric function and experimental results.

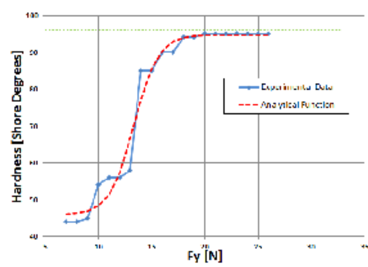

Figure 3. Evolution of hardness depending on the medio-lateral component, Fy

A similar shape may be obtained using a decimal logarithm:

$$Y = LowerLimit + (UpperLimit - LowerLimit) / \{1 + 10^{[(Log(a) - X)]}\} \quad (9)$$

It is noted that this new formula has fewer variables that can be stored, which leads to a simpler mathematical model. Although experimental data fit is not as good as in the case of the sigmoidal, the simplicity of the equation may be helpful in further research. Thus, the constants obtained through regression lead to the final shape (10):

$$Hardness = 49.36 + \frac{44}{1 + 10^{13.5 - x}} \quad (10)$$

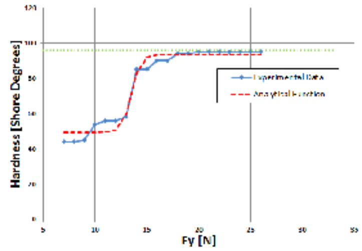

Figure 4. Evolution of hardness depending on the medio-lateral component, Fy

The figure above illustrates the correlation between experimental data and the empirical logarithmic relationship derived by nonlinear regression.

A general conclusion is that all three models of empirical equations have a horizontal asymptote corresponding entirely with experimental data. This, coupled with the step-like shape of the data set, leads to the conclusion that each of the models fulfills its purpose to describe the physical model (to the extent it is studied).

Only by further experimental analysis we can deduce which the equations found is correct one. This can be determined only by isolating the different terms in each equation and their correlation with physical parameters of the experiment.

CONCLUSIONS

The aim of this study is to determine the variation of the hardness of the material used to manufacture the shoe heel, depending on the medio-lateral component, Fy, of the ground reaction force.

It was shown that there is the possibility of changing the distribution of ground reaction force components by heel construction and by using materials of appropriate hardness.

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