OFF-AXIS MECHANICAL PROPERTIES OF FRP COMPOSITES

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Composite structures made of fibre reinforced polymer (FRP) composites are usually built-up of several individual unidirectional laminas which may have their natural material axes at different orientations with respect to the loading direction. Off-axis mechanical properties of the unidirectional FRP lamina can be determined either experimentally or predicted theoretically. One way to theoretically predict the off-axis stiffness and strength properties of a unidirectional orthotropic lamina is by applying the macromechanical concepts. This paper presents the available macromechanical approaches utilized to calculate the off-axis stiffness and strength properties of a unidirectional orthotropic lamina for which the loading directions are different from the principal material axes. In addition, a case study is presented, in order to apply the macromechanical tools to a FRP lamina made of glass fibres and epoxy matrix.

Keywords: FRP composite lamina, off-axis strength properties, off-axis stiffness properties.

INTRODUCTION

From macromechanical point of view, the off-axis mechanical properties of the unidirectional FRP composites are anisotropic, due to their variation with respect to the orientation of the reference plane. The aim of the macromechanical approach is to correlate the stiffness and strength properties along an arbitrary direction with the basic properties of the unidirectional FRP composite referred to its principal material directions (Daniel and Ishai, 2006). FRP composite laminates consist of two or more laminas, bonded together so that they can act as integral structural elements (Agarwal *et al.*, 2006). For this reason the understanding of the individual lamina characteristics should precede the analysis of the laminated structures theory.

Orthotropic Laminas

A lamina or a ply consists of a flat or curved arrangement of unidirectional fibers embedded in a support matrix and it represents the basic element of a composite material. For the orthotropic lamina, the material axes are perpendicular and stand as symmetry planes.

Generally Orthotropic Lamina

The generally orthotropic lamina is that for which the material axes (1, 2) do not coincide with the global coordinates axes (x,y), that may be the axes of the loading directions (Barbero, 2011). The material axes are rotated with respect to the reference system by angle Θ, as presented in Figure 1.

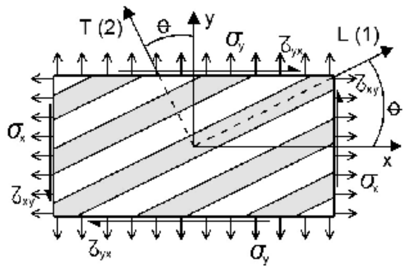

Figure 1. Generally orthotropic lamina

The constitutive equations for the generally orthotropic lamina are presented in Equations 1 and 2.

$$\{\sigma_i\} = \left[\overline{Q}_{ij}\right]\{\varepsilon_i\} \tag{1}$$

$$\{\varepsilon_i\} = \left[\overline{S}_{ij}\right]\{\sigma_i\} \tag{2}$$

where, $\{\sigma_i\}$ and $\{\varepsilon_i\}$ are the components of the stress and strain matrices, respectively. The matrices $\left[\overline{Q}_{ij}\right]$ and $\left[\overline{S}_{ij}\right]$ are the reduced transformed stiffness and compliance matrices, respectively. The elements $\overline{Q}_{ij}$ and $\overline{S}_{ij}$ are functions of the elastic properties of the lamina along its principle axes (1,2) and of the fibre orientation angle, Θ.

Stiffness Properties

Axial Modulus of Elasticity, E_x

Assuming that the only nonzero stress component acting on the lamina is σ_x, the axial modulus of elasticity (E_x) can be expressed in terms of the engineering constants in the principal material coordinates and of the fibre orientation Θ.

$$E_x = \frac{1}{\frac{1}{E_1}c^4 + \left(\frac{1}{G_{12}} - 2\frac{\nu_{12}}{E_1}\right)s^2c^2 + \frac{1}{E_2}s^4} \tag{3}$$

where, E_1, E_2 and G_{12} are the axial and shear moduli of elasticity in the principal material axes, ν_{12} is the first Poisson's coefficient and the trigonometric functions *sinΘ* and *cosΘ* are denoted with *s* and *c*, respectively.

The variation of the axial modulus of elasticity is presented in Figure 2. It can be seen that the values of E_x decrease as the angle between the material axes and the global coordinates axes increases, between E_1 and E_2.

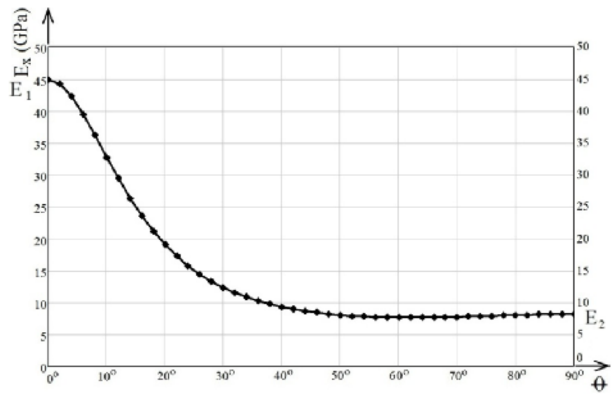

Figure 2. Variation of E_x with respect to Θ

Axial Modulus of Elasticity, E_y

Imposing that the only stress component different from zero is σ_y, the axial modulus of elasticity E_y can be also expressed with respect to the fibre inclination angle Θ and to the elastic properties of the lamina along its principal axis.

$$E_y = \frac{1}{\frac{1}{E_1}s^4 + \left(\frac{1}{G_{12}} - 2\frac{\nu_{12}}{E_1}\right)s^2 c^2 + \frac{1}{E_2}c^4} \tag{4}$$

Figure 3 presents the variation of E_y with respect to angle Θ. Unlike the case of the longitudinal modulus of elasticity in x direction, the interval between 0° and 60° is characterized by a smaller rate of increase while in the 60°-90° interval, E_y has the highest rate of increase. Appling Equation 4 for Θ = 0° and Θ = 90°, E_y equals E_2 and E_1, respectively.

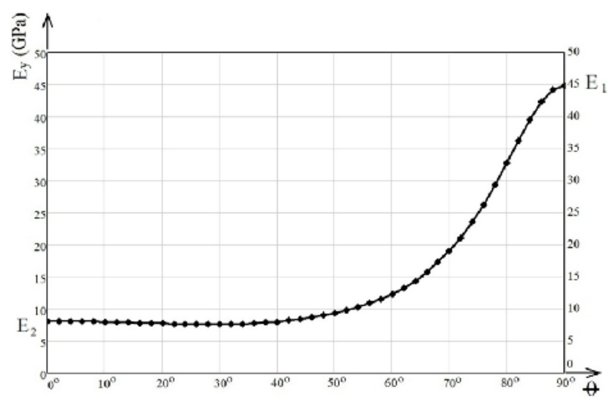

Figure 3. Variation of E_y with respect to Θ

Shear Modulus of Elasticity, G_{xy}

The shear modulus of elasticity can be calculated under the assumption of pure shear state of stress. In this case, the only non-zero stress component is τ_{xy}; the shear modulus of elasticity, G_{xy} can be also expressed as a function and of the elastic properties of the lamina in its principal directions and of the fibres inclination angle.

$$G_{xy} = \frac{1}{2\left(\frac{2}{E_1}+\frac{2}{E_2}+\frac{4\nu_{12}}{E_1}-\frac{1}{G_{12}}\right)s^2c^2+\frac{1}{G_{12}}(s^4+c^4)} \tag{5}$$

The variation of the shear modulus of elasticity is presented in Figure 4. It can be seen that G_{xy} has the highest values when Θ is 45° while G_{xy} equals G_{12} when Θ is 0° or 90°.

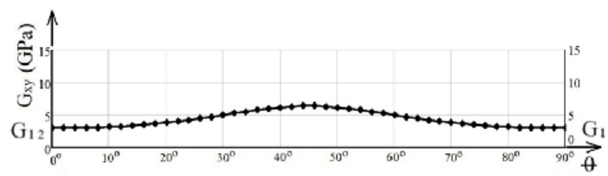

Figure 4. Variation of G_{xy} with respect to Θ

Poisson's Ratios, ν_{xy}, ν_{yx}

The first Poisson's ratio ν_{xy}, can be calculated considering that the only nonzero stress component is σ_x (Equation 6), while the second Poisson's ratio ν_{yx}, can be obtained when σ_y is different from zero, (Herakovich, 1998), (Equation 7).

$$\nu_{xy} = \frac{\left[c^2s^2\left(1+\frac{E_1}{E_2}-\frac{E_1}{G_{12}}\right)-(c^4+s^4)\nu_{12}\right]}{\left[c^4+c^2s^2\left(-2\nu_{12}+\frac{E_1}{G_{12}}\right)+s^4\,\frac{E_1}{E_2}\right]} \tag{6}$$

$$\nu_{yx} = \frac{\left[c^2s^2\left(1+\frac{E_1}{E_2}-\frac{E_1}{G_{12}}\right)-(c^4+s^4)\nu_{12}\right]}{\left[s^4+c^2s^2\left(-2\nu_{12}+\frac{E_1}{G_{12}}\right)+c^4\,\frac{E_1}{E_2}\right]} \tag{7}$$

Figure 5 presents the variation of ν_{xy} and ν_{yx} with respect to the inclination angle of the fibres, Θ. The first Poisson's ratio has the highest values when Θ is 29° and equals ν_{12} or ν_{21} when Θ is 0° or 90°, respectively. Similarly, the second Poisson's ratio has the same values as ν_{12} or ν_{21} when the inclination angle of the fibres is 0° or 90° but ν_{yx} reaches its maximum value when Θ is 61°.

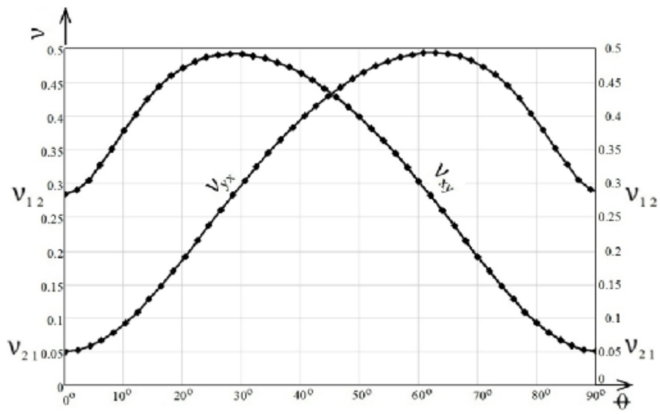

Figure 5. Variation of ν_{xy} and ν_{yx} with respect to Θ

Strength Properties

Off-axis Tensile Strength

The maximum tensile strength along any direction can be calculated with Equation 8 which is derived from the Tsai-Hill failure criterion (Kaw, 2005).

$$f_{x(\Theta)t} = \frac{1}{\sqrt{\frac{c^4}{f_{Lt}^2} + \frac{s^4}{f_{Tt}^2} + c^2 s^2 \left(\frac{1}{f_{Lts}^2} - \frac{1}{f_{Lt}^2} \right)}} \tag{8}$$

where, f_{Lt} and f_{Tt} are the longitudinal and transverse tensile strength of the lamina along its principle directions and f_{LTs} is the in-plane shear strength of the lamina.

CASE STUDY

Determine the mechanical properties in the global coordinates system (x,y) of the following 45° angle unidirectional E glass / Epoxy composite (Taranu et. al., 2013). The properties of the lamina along its principal axes are: f_{Lt} = 900 MPa, f_{Tt} = 19.5 MPa, f_{LTs} = 25.9 MPa, E_1 = 44.30 GPa, E_2 = 6.77 GPa, G_{12} = 2.95 GPa, ν_{12} = 0.278 and ν_{21} = 0.053.

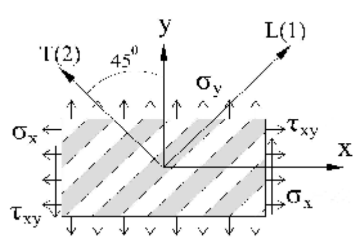

Figure 6. 45° angled unidirectional E glass / Epoxy composite lamina

Because Θ = 45° (s = c) it results that $E_x = E_y$ and $\nu_{xy} = \nu_{yx}$.

$$E_x = \frac{1}{\frac{1}{E_1}c^4 + \left(\frac{1}{G_{12}} - 2\frac{\nu_{12}}{E_1}\right)s^2\,c^2 + \frac{1}{E_2}s^4} = 8052.9MPa = E_y \tag{3}$$

$$G_{xy} = \frac{1}{2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s^2c^2 + \frac{1}{G_{12}}(s^4 + c^4)} = 5469.4MPa \tag{5}$$

$$\nu_{xy} = \frac{\left[c^2s^2\left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right) - (c^4 + s^4)\nu_{12}\right]}{\left[c^4 + c^2s^2\left(-2\nu_{12} + \frac{E_1}{G_{12}}\right) + s^4\,\frac{E_1}{E_2}\right]} = 0.365 = \nu_{yx} \tag{6}$$

$$f_{x(\Theta)t} = \frac{1}{\sqrt{\frac{c^4}{f_{Lt}^2} + \frac{s^4}{f_{Tt}^2} + c^2s^2\left(\frac{1}{f_{Lts}^2} - \frac{1}{f_{Lt}^2}\right)}} = 31.15MPa \tag{8}$$

CONCLUSION

This paper presents the macromechanical approach that can be applied to determine the off-axis stiffness and strength properties of FRP composite laminas. These theoretical methods of predicting the properties of an FRP product subjected to a certain state of stress having reference directions different from the materials principal ones, can turn to be effective not only from the economical point of view but also from the time consuming one.

Experimental determinations for various inclination angles (Θ) of the fibers orientation are prohibitive and difficult to be carried out. Moreover, the off-axis properties of an FRP composite lamina should be previously determined by theoretical approaches followed by selective experimental tests aiming to validate these results.

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