# CHARACTERIZATION OF THE STATE PLANAR ORIENTATION FOR SHORT NATURAL FIBER IN POLYMERIC COMPOSITES BY MEANS OF THE TENSOR ORIENTATION

JORGE ISAAC FAJARDO SEMINARIO[1], GUSTAVO SUAREZ[2], LUIS JAVIER CRUZ[2], LUIS EDUARDO GARZÓN MUÑOZ[1], LUIS MARCELO LÓPEZ[1]

[1] *Universidad Politécnica Salesiana, Calle Vieja12-30 y Elia Liut, Cuenca, Ecuador, jfajardo@ups.edu.ec*
[2] *Universidad Pontificia Bolivariana, Facultad de ingeniería Mecánica, Campus de Laureles Circular 1, 70-01, Medellín Colombia, luis.cruz@upb.edu.co*

This investigation presents a detailed description to evaluate and predict the orientation state of short fibers of "Guadua Angustifolia Kunth" (GAK) as reinforcement in polypropylene (PP) matrix by using the second order orientation tensor. For this, several samples were prepared by injection molding and then metallographically polished. The aim of this work is to predict the state planar orientation for short natural fibers as an initial stage to determine the mechanical behavior of composite materials. The tensor orientation is determined by using digital image processing in selected polished sections of different material samples. For both digital images processing and computational modeling it was possible to find different orientation states over all specimens. This method will allow us to know the real effect of the flow fibers on the main mechanical properties of the biocomposites.

Keywords: Orientation tensor, polymeric composites, image processing.

## INTRODUCTION

The averaging method used (Lagzdins *et al.*, 2009) allows for the estimation of elastic properties of a polymeric composite reinforced by short randomly oriented fibers. Such elastic properties were also studied by (Modniks and Andersons, 2010) using the unit cell properties consisting of a fiber of an average length and matrix according to the fiber volume fraction by means of orientation averaging. The stiffness tensor components $C_{ijkl}$ of the composite are calculated by using the formula (Modniks *et al.*, 2011):

$$C_{ijkl} = \iint C^*_{ikjl}(\theta,\phi)\psi(\theta,\phi)\sin\theta\, d\theta\, d\phi \qquad (1)$$

$C^*_{ijkl}$ are the stiffness tensor components of the unit cell, $\psi(\theta,\phi)$ the distribution density of the fiber orientation, $\theta$ and $\phi$ are the elevation and azimuthal angles of a fiber.

The present investigation shows the characterization of the orientation state of short fibers using the orientation tensor. The first approximation used the distribution density of the fiber orientation $\psi(\theta,\phi)$ and second one by means of orientation tensor supported with digital processing images.

## THEORETICAL DESCRIPTION

As described (Advani and Tucker, 1987) in earlier reports and (Modniks *et al.*, 2011) recently, the fibers are assumed as rigid cylinders, with an uniform cross-section area and constant length. With these assumptions, the fiber orientation state can be studied by using the probability distribution function for orientation $\psi(\theta,\phi)$. This function defines the probability of finding a fiber between angles $\theta_1$ and $\theta_1 + d\theta$ and $\phi_1$ and $\phi_1 + d\phi$ and described as:

$$P\left(\theta_1 \leq \theta \leq \theta_1 + d\theta, \phi_1 \leq \phi \leq \phi_1 + d\phi\right) = \psi(\theta_1, \phi_1) \sin\theta_1 \, d\theta \, d\phi \tag{2}$$

The orientation can be described by associating a unit vector **p** with the single fiber, as shown in figure 1. Thus, the distribution function can be written as a function of **p** vector, $\psi(\vec{p})$.

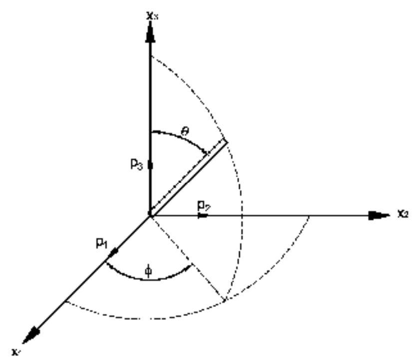


Figure 1. Spherical coordinates system for describing of a single fiber

In according to figure 1, the components of unit vector **p** are related to elevation and azimuthal angles as:

$$p_1 = \sin\theta\cos\phi \qquad p_2 = \sin\theta\sin\phi \qquad p_3 = \cos\theta \tag{3}$$

By denoting the fixed length of the vector **p**, ( $p_i p_i$ =1), the set of all possible directions of the vector corresponds to unit sphere. The integral over all possible directions over the surface of the unit sphere can be calculated by

$$\oint d\vec{p} = \int_0^{2\pi}\int_0^{\pi} \psi(\theta,\phi)\sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi \tag{4}$$

The function $\psi(\theta,\phi)$ must satisfy some conditions such as, periodicity, normalization and continuity condition. To denote the orientation state of short fibers in composites, robust processing conditions which is computationally cumbersome is required. Taking into account the symmetrical condition of the distribution function, one set of orientation tensors can be defined by forming dyadic products of the vector **p** (Advani and Tucker, 1987). By integrating dyadic products with the distribution functions only even order orientation tensors are calculated and being the major interest is the second and fourth order orientation tensors.

$$a_{ij} = \oint p_i p_j \psi(\vec{p}) d\vec{p} \qquad a_{ijkl} = \oint p_i p_j p_k p_l \psi(\vec{p}) d\vec{p} \tag{5}$$

By expanding the equation (5), in particular the second order tensor for i,j=1,2 y 3, a symmetrical matrix is obtained. Due to the normalization condition, the second order tensor trace is equal to unity, in other words the summation of the orientation percentages must be 100%. By considering real composites, each discrete fiber sample is measured where the components of the tensors are calculated by a summation, namely (Lee *et al.*, 2002),

$$a_{ij} = \frac{\sum(p_i p_j) L_n F_n'}{\sum L_n F_n'} \tag{6}$$

where $F_n'$ represents the weighting function for the *n*th fiber,

A planar orientation state considers that all fibers lie on a single plane. This fact is being widely used to study composite materials due to their approximated planar orientation (Advani and Tucker, 1987). The planar orientation of the second order tensor can be defined as,

$$a_{ij} = \int_0^{2\pi} \psi_\psi(\phi) p_i p_j d\phi \tag{7}$$

The planar orientation of the second order tensor is used to describe the orientation state in the main flow direction. By taking the summation for N fibers, $a_{ij}$ is to re-write as follows (Eberhardt and Clarke, 2001),

$$a_{ij} = \frac{1}{N}\left(\sum_{n=1}^{N} p_i^n p_j^n\right) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

(8)

| | |
|---|---|
| $a_{11} < 0.35$ | is considered perpendicular to the flow direction. |
| $a_{11} > 0.7$ | is considered parallel to the flow direction. |
| $0.5 < a_{11} < 0.6$ | is considered a random orientation. |

By considering the planar orientation state, the main components of the tensor for the axis $x_1$ and $x_2$, as shown in figure 1, can be determined by

$$a_{11} = \frac{1}{N}\sum_{n=1}^{N} cos^2 \varphi \qquad a_{12} = \frac{1}{N}\sum_{n=1}^{N} \cos\varphi sin\varphi \tag{9}$$

where N is the total number of fibers in the composite, $\varphi$ is the planar orientation angle of each fiber. To calculate the preferred angle of the fibers Yasuda et al. (2004) used the following equation,

$$\tan 2\alpha = \frac{2a_{12}}{a_{11} - a_{22}} \tag{10}$$

Unlike Yasuda's results and depicted in figure 2, the orientation state is obtained in this report taking into account the $a_{11}$ element of the equation (8) for the two-dimensional case.

## EXPERIMENTAL

The orientation tensor method was employed to evaluate the orientation state of short fibers using a micrograph of a polymeric compound of polypropylene and short fibers of GAK (PP+GAK). Homopolymer Polypropylene was supplied by Braskem H-306 and bamboo fiber was supplied by Bamboo House in Ecuador. Fiber lengths of about 4mm and diameters of 0.2mm according to ASTM E11-95 for mesh 45 were used. The bamboo fibers were chemically modified using NaOH at 5%.

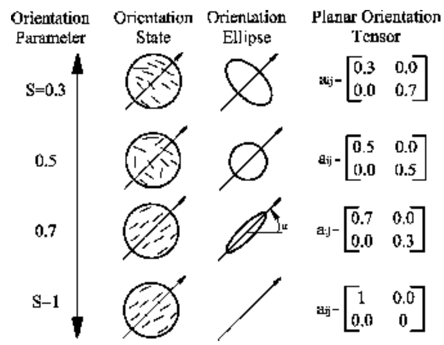


Figure 2. Relation between the orientation parameter, orientation state, orientation ellipse and the planar orientation tensor are shown

ASTM D 638 samples were obtained by injection using a Dr. Boy 35 horizontal injection moulding machine. Metallographic polish was performed on each sample and then unsaturated polyester resin was embedded. Micrograph imaging were recorded by means of a light Olympus BX-41M (5X) microscope. Different positions of the sample were studied by sectioning the sample at 10, 40 and 70 mm along the flux axis. Furthermore, several samples were analyzed at different cutting depths (1, 2 and 3 mm), as shown in figure 3.

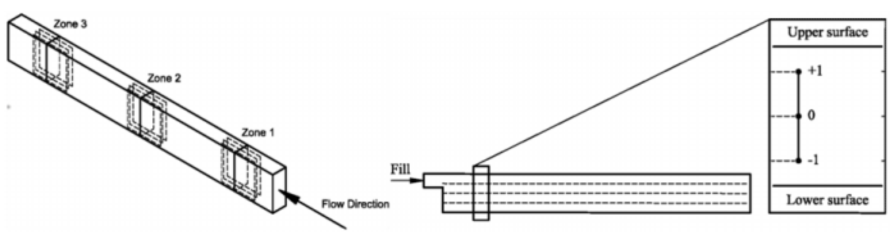


Figure 3. Sectioning of samples along and across the main flux axis

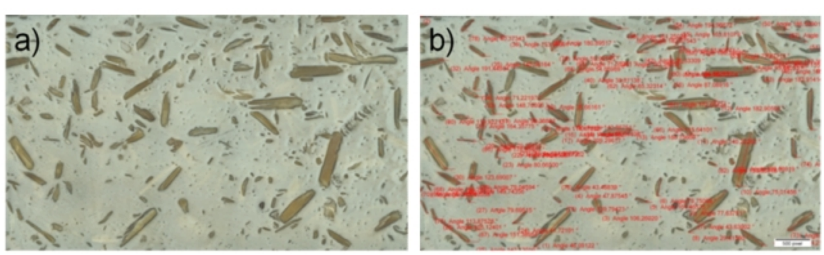


Figure 4. Micrograph recorded in a) zone 1 at 1mm in depth and b) measurements of φ angle

Measurements of the fiber orientation were carried out using imaging processing Olympus Stream® software. Orientation values of each fiber along the main flux axis were measured. Figure 4 shows orientation angles in the plane (φ) of the zone 1 with a depth of 1mm. The measured values from imaging processing were used to obtain the orientation tensor of GAK fibers by means of a MatLab R2012a® script.

## RESULTS

Figure 5 shows the trend of the state of orientation ($a_{11}$) and the preferred angle (α) of the samples with 30% GAK along the flow direction and through thickness z. From the $a_{11}$ calculated element by the second order orientation tensor applied to each one of the areas in each specimen with respect to the direction of entering flow a high orientation of the fibers was measured. The main element of the tensor orientation is above 0.8 for each value of z. Further guidance can be seen in the regions near the injection point. Asymmetry was also observed with respect to the thickness. Generally highly oriented zones are those regions in which the polymer matrix solidifies first, with slight variations due to the forces produced by front advance of the flow. Coincidence between the orientation state using the tensor orientation and the maximum eigenvalue which also defines the state was observed. The calculated error reaches the 0.82% which provides reliable results with the application of both methods. Table 1 summarizes the values obtained for the zone 1.

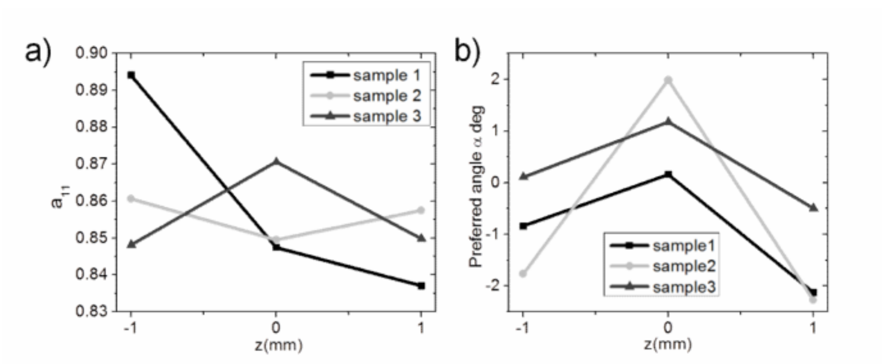


Figure 5. Distributions of the orientation state by means of $a_{11}$ and the preferred angle in the main direction flux respect to z depth are shown

Figure 5b shows the distribution of the preferred angle of orientation and its evolution with the thickness of the specimens. Similar results were found for other preparations (40% wt/wt). The preferred angle of the fiber measured with respect to the direction of flow is very narrow and is in the range of ±2 grades. It shows a fairly symmetrical distribution of preferential angle and high deviations in the center of the flow. This effect is due to the insulating nature of the polymers having the specimens which take more time to solidify in this region, so the fibers have more mobility and are more dispersed.

Table 1. Comparison between calculated values $a_{11}$ and the order parameter S of the planar orientation state in the zone 1 of the specimen (30% GAK) are shown

| | A1_1mm_30% | A1_2mm_30% | A1_3mm_30% |
|---|---|---|---|
| $a_{11}$ | 0.8232 | 0.8237 | 0.8705 |
| S (order parameter) | 0.8234 | 0.8241 | 0.8777 |
| Error | 0.024% | 0.049% | 0.827% |

## CONCLUSIONS

The orientation tensor method has been used to evaluate the orientation state of short fibers in polymeric composites without dependence of the aspect ratio of fibers. Both orientation tensor and imaging processing technique were employed to calculate the orientation state of fibers in composites. It was possible to compare results obtained using the first element of the orientation tensor $a_{11}$ and the higher eigenvalue of each orientation state. Unlike Yasuda's results depicted in figure 2, the orientation state in this report was calculated by taking into account the $a_{11}$ element of the equation (8) for the two-dimensional case.

The central portion of the samples loaded with 30 and 40% of GAK showed greater dispersion angles and lesser preferred orientation state due to the time taken for this region to solidify. However it offers the possibility of experiencing the fiber orientation through the application of shear that will control the orientation state in that area and to infer on their future mechanical properties. The difference in results between the analysis through the main element of the tensor orientation and the largest eigenvalue is minimal and reaches the 0.82% allowing for the estimation of orientation state by any of the methods described with complete reliability.

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